UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT:	**PURE AND APPLIED MATHEMATICS**
MODULE:	**APM2A10** INTRODUCTION TO DIFFERENTIAL EQUATIONS
CAMPUS:	**AUCKLAND PARK KINGSWAY**

SUPPLEMENTARY EXAMINATION

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ASSESSORS	**MR JM HOMANN** **DR MV VISAYA**
MODERATOR	**PROF M KHUMALO**
DURATION: 2 HOURS	**MARKS: 50**

NUMBER OF PAGES	1 COVER PAGE 2 QUESTION PAGES 1 FORMULA PAGE
INSTRUCTIONS	ANSWER ALL THE QUESTIONS. SHOW ALL CALCULATIONS. POCKET CALCULATORS MAY BE USED. SYMBOLS HAVE THEIR USUAL MEANING.

QUESTION 1 [5 MARKS]

Consider the differential equation (DE)

$$y' = \frac{1+y}{x}.$$

a) On the same x, y-axes, sketch a slope field for the given differential equation at the points (0,1), (1,0), (0,0), (-1,0), (0,-1), (1,1), (-1,-1), (-1,1) and (1,-1).

b) Solve the DE and denote the constant by C. Sketch three curves associated to $C > 0, C = 0$, and $C < 0$.

QUESTION 2 [10 MARKS]

Use the method of undetermined coefficients to determine a particular solution of

$$y'' + 3y' + y = (2 - 6x)\cos x - 9\sin x.$$

QUESTION 3 [10 MARKS]

Make use of the unit step function to determine the Laplace transform of $f(t)$ for $t \geq 0$, and where

$$f(t) = \begin{cases} \sin t, & 0 \leq t < \frac{\pi}{2}, \\ \cos t - 3\sin t, & \frac{\pi}{2} \leq t < \pi, \\ 3\cos t, & t \geq \pi. \end{cases}$$

QUESTION 4 [15 MARKS]

Use the Laplace transform to solve the initial value problem

$$y'' + 2y' + 2y = 1,$$

with $y(0) = -3$ and $y'(0) = 1$. Verify that your answer satisfies the initial conditions.

QUESTION 5 [5 MARKS]

A 64 kg weight is suspended from a spring with stiffness 25 N/m. It is initially displaced 18 meters above equilibrium and released from rest. Set-up only the equation of motion for the spring system. Moreover, determine what sort of damping (critical, over- or under-damping) is experienced.

QUESTION 6 [5 MARKS]

Using the definition of the Laplace transform of a function, evaluate $\mathcal{L}(f)$ for $f = te^{-t}$.

Information

$$my'' = -mg - cy' + k\Delta L - ky + F$$

$$\cos\left(t + \frac{\pi}{2}\right) = -\sin t \qquad \sin\left(t + \frac{\pi}{2}\right) = \cos t$$

$$\cos(t + \pi) = -\cos t \qquad \sin(t + \pi) = -\sin t$$

$$\sin kt = \mathcal{L}^{-1}\left(\frac{k}{s^2 + k^2}\right)$$

$$\cos kt = \mathcal{L}^{-1}\left(\frac{s}{s^2 + k^2}\right)$$

$$e^{at} = \mathcal{L}^{-1}\left(\frac{1}{s - a}\right)$$

Theorem

$$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$$

and

$$\mathcal{L}(f'') = s^2\mathcal{L}(f) - f'(0) - sf(0)$$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$u(t - \tau) = \begin{cases} 0 & t < \tau \\ 1 & t \geq \tau \end{cases}$$

Theorem
Let g be defined on $[0, \infty)$. Suppose $\tau \geq 0$ and $\mathcal{L}\left(g(t + \tau)\right)$ exists for $s > s_0$. Then $\mathcal{L}\left(u(t - \tau)g(t)\right)$ exists for $s > s_0$, and

$$\mathcal{L}(u(t - \tau)g(t)) = e^{-s\tau}\mathcal{L}\left(g(t + \tau)\right)$$

Definition A function f is said to be of exponential order s_0 if there are constants M and t_0 such that

$$|f(t)| \leq Me^{s_0 t}, \quad t \geq t_0.$$

END OF QUESTION PAPER